EM Lecture 5 – worked examples

Q1) A dielectric material has a uniform polarisation $\underline{P} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ Cm^{-2}. A 0.2 m^2 surface of this material has a normal vector in the $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ direction. What is the total polarisation charge on this surface?

A1) The polarisation surface charge density is given by $\sigma_b = \underline{P} \cdot \hat{\underline{n}} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \frac{1}{3} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = -1/3\,\mathrm{Cm}^{-2}$ (where we have normalised the vector perpendicular to the surface). Hence the total charge on this surface is

0.2 x –1/3 = -0.067 C.

Q2) The polarisation of a dielectric is given by $\underline{P} = A\left(x^2 \hat{\underline{i}} - z \hat{\underline{k}} \right)$ where A is a constant. What is the value of the polarisation volume charge density at the point (1,2,-3) m?

A2) The polarisation volume charge density is given by

$$\rho_b = -\nabla \cdot \underline{P} = -A \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} x^2 \\ 0 \\ -z \end{pmatrix} = A(-2x+1) = A(-2\times 1+1) = \underline{\underline{-A}} \text{ in this case.}$$

Q3) On the surface of a sphere of radius R the surface polarisation is given by $P(t) = P_o \cos(\omega t)$ in the direction of the outward normal to the surface. P_o is a constant. Obtain an expression for the total current flowing through the surface of the sphere as a function of time.

A3) We have that the polarisation current density is given by:

$$J_b(t) = \frac{\partial P}{\partial t} = -P_o \omega \sin(\omega t)$$

flowing radially outwards. With a total surface area $4\pi R^2$ the current flowing through the surface is

$$\underline{\underline{I_b(t) = -4\pi R^2 P_o \omega \sin(\omega t)}}$$