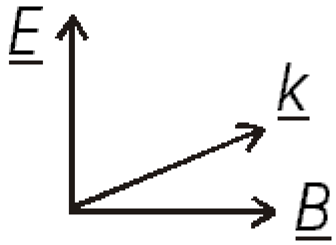
EM Lecture 4 – worked examples



Q1) An EM wave is travelling in the $\begin{pmatrix}2\\1\\4\end{pmatrix}$ direction. Give any consistent directions for $\underline{E}_o$ and $\underline{B}_o$. If E_o = 5 Vm^{-1} what are the corresponding actual $\underline{E}_o$ and $\underline{B}_o$?

A2) $\underline{E}$, $\underline{B}$ and $\underline{k}$ are mutually perpendicular in the sense of the diagram shown above. To obtain, say, $\underline{E}_o$, the direction of $\underline{E}$ – field polarisation must be consistent with $\underline{k}.\underline{E}_o = 0$. By inspection, one suitable solution (there are an infinite number) is that $\underline{E}_o$ is in the direction of $\begin{pmatrix}1\\-2\\0\end{pmatrix}$. $\underline{B}_o$ must then be in the direction of $\underline{k}\times\underline{E}_o$ or

$$\begin{pmatrix}2\\1\\4\end{pmatrix}\times\begin{pmatrix}1\\-2\\0\end{pmatrix}=\begin{pmatrix}8\\4\\-5\end{pmatrix}.$$

If $\underline{E}_o$ = 5 Vm^{-1} we must normalise as follows:

$$\underline{E}_o = \frac{5}{\sqrt{1^2+2^2+4^2}}\begin{pmatrix}1\\-2\\4\end{pmatrix}\text{Vm}^{-1} = \underline{\underline{1.09\begin{pmatrix}1\\-2\\4\end{pmatrix}\text{Vm}^{-1}}}\,.$$

Knowing $B_o = E_o/c$ this gives $\underline{B}_o = \dfrac{5}{3\times10^8\times\sqrt{8^2+4^2+5^2}}\begin{pmatrix}8\\4\\-5\end{pmatrix}\text{T} = \underline{\underline{1.63\times10^{-9}\begin{pmatrix}8\\4\\-5\end{pmatrix}\text{T}}}\,.$

Q2) If $\underline{B}_o$ is in the $\begin{pmatrix}1\\1\\1\end{pmatrix}$ direction and $\underline{E}_o$ is in the $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ direction what is the direction of $\underline{k}$?

A2) $\underline{k}$ is in the direction of $\underline{E}_o$ x $\underline{B}_o$ (see diagram) i.e. in the direction of

$$\begin{pmatrix}1\\0\\-1\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix}=\underline{\underline{\begin{pmatrix}1\\-2\\1\end{pmatrix}}}$$

Q3) A 3 MHz EM wave has wavevector $\underline{k}$ in the direction $\begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$. What is the actual wavevector $\underline{k}$?

A3) In free space we have that $\frac{\omega}{k} = c$ so in this case

$$k = \frac{\omega}{c} = \frac{3 \times 2\pi \times 10^6}{3 \times 10^8} = 2\pi \times 10^{-2} \text{ m}^{-1}.$$

We can properly normalise $\underline{k}$ using

$$\underline{k} = \frac{2\pi \times 10^{-2}}{\sqrt{1^2 + 3^2 + 4^2}} \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \text{m}^{-1} = 0.0123 \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \text{m}^{-1}$$