EM Lecture 13 – worked examples

Q1) A plane wave in air polarised normal to the plane of incidence has an electric field amplitude $E_o = 5\times10^{-4}$ Vm^{-1} and is incident at an angle of 35 degrees on the interface with a non-magnetic LIH material with a refractive index of 2.6. What is the amplitude of the E – field of the reflected wave?

A2) We can compute the E – field of the reflected wave using the appropriate Fresnel equation for $\underline{E}$ polarised normal to the plane of incidence:

$$\frac{E_{OR}}{E_{OI}} = \frac{\frac{n_1}{n_2}\cos\theta_I - \cos\theta_T}{\frac{n_1}{n_2}\cos\theta_I - \cos\theta_T}$$

We first need to find θ_T. Using Snell's Law with $n_1 = 1$ and $n_2 = 2.6$ we have that

$$\frac{n_1}{n_2} = \frac{\sin\theta_T}{\sin\theta_I} \therefore \sin\theta_T = \frac{n_1}{n_2}\sin\theta_I = \frac{1}{2.6}\sin 35° = 0.2206$$

$$\Rightarrow \theta_T = \sin^{-1} 0.2206 = 12.74° \text{ and } \cos\theta_T = \cos 12.74° = 0.9754.$$

Using $\cos\theta_I = \cos 35° = 0.8192$ we have that

$$E_{OR} = 5\times10^{-4}\,\frac{\frac{1}{2.6}\times0.8192 - 0.9754}{\frac{1}{2.6}\times0.8192 + 0.9754} = -2.56\times10^{-4}\ \text{Vm}^{-1}.$$

The negative sign indicates that the reflected wave is π out of phase with the incident wave, which we would expect given that $n_2 > n_1$, so the amplitude of the reflected wave is just $\underline{\underline{2.56\times10^{-4}\ \text{Vm}^{-1}}}$.

Q2) How exactly can polarised light be produced by using a block of non-magnetic LIH material with refractive index n = 3.5?

A2) Reflect a wave of general polarisation off the block (n_2 = 3.5) in air (n_1 = 1).

The Brewster angle is given by $\tan\theta_{IB} = \frac{n_2}{n_1} \ \Rightarrow \theta_{IB} = \tan^{-1}\frac{n_2}{n_1} = \tan^{-1}\frac{3.5}{1} = 74.1°$

The reflected wave will be polarised entirely normal to the plane of incidence.

Q3) Imagine that there are two non-conducting LIH materials such that one has $\mu = 2.0\times10^{-6}$ Hm^{-1} and $\varepsilon = 5.0\times10^{-11}$ Fm^{-1} and the other has $\mu = 1.25\times10^{-6}$ Hm^{-1} and $\varepsilon = 2.0\times10^{-11}$ Fm^{-1}. What is the reflection coefficient at normal incidence at the planar interface between these two materials?

A3) We must use the appropriate Fresnel equation at normal incidence:

$$\frac{E_{OR}}{E_{OI}} = \frac{\frac{n_1}{\mu_1} - \frac{n_2}{\mu_2}}{\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}} = \frac{\frac{n_1}{n_2} - \frac{\mu_1}{\mu_2}}{\frac{n_1}{n_2} + \frac{\mu_1}{\mu_2}}.$$

(Just use the general Fresnel equations expression and put $\cos\theta_I = \cos\theta_T = 1$.)

The refractive indices of the two materials are given by:

$$n = \frac{ck}{\omega} = \frac{c}{v_{ph}} = c\sqrt{\mu\varepsilon}$$

For the first material $n_1 = 3\times10^8 \times \sqrt{2.0\times10^{-6}\times5.0\times10^{-11}} = 3$

and for the second material we have $n_2 = 3\times10^8 \times \sqrt{1.25\times10^{-6}\times2.0\times10^{-11}} = 1.5$

Thus we have that $\frac{n_1}{n_2} = \frac{3.0}{1.5} = 2$ and $\frac{\mu_1}{\mu_2} = \frac{2.0\times10^{-6}}{1.25\times10^{-6}} = 1.6$.

Hence, $R = \left(\frac{E_{OR}}{E_{OI}}\right)^2 = \left(\frac{2-1.6}{2+1.6}\right)^2 = \underline{\underline{0.012}}$

In practice, it does not matter which material is on which side of the interface as the reflection coefficient will still be the same if the values of $\frac{n_1}{n_2}$ and $\frac{\mu_1}{\mu_2}$ are inverted to 1/2 and 1/1.6 respectively.