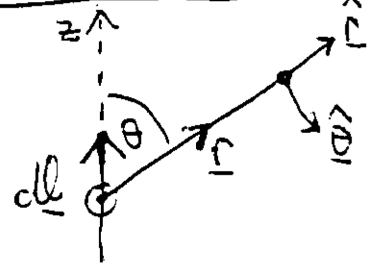
EM Answer to Q9



$$d\underline{l} = \left[\cos\theta\,\hat{\underline{r}} - \sin\theta\,\hat{\underline{\theta}}\right] dl$$

$\Rightarrow$ no $\hat{\phi}$ component

For the purposes of this question we may take

$$\frac{\mu_0 I}{4\pi} e^{j\omega t} dl = C \text{, a constant.}$$

Thus $\underline{B} = \nabla\times\underline{A} = \nabla\times\left[\frac{\mu_0 I}{4\pi r} e^{j\omega(t-\frac{r}{c})} d\underline{l}\right]$

$$= C.\nabla\times\left[\frac{e^{-\frac{j\omega r}{c}}}{r}\left(\cos\theta\,\hat{\underline{r}} - \sin\theta\,\hat{\underline{\theta}}\right)\right]$$

$$\nabla\times\underline{A} = \frac{C}{r^2\sin\theta}\begin{vmatrix} \hat{\underline{r}} & r\hat{\underline{\theta}} & r\sin\theta\,\hat{\underline{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

3 marks for correctly identifying components

where $A_r = \frac{e^{-\frac{j\omega r}{c}}}{r}\cos\theta$, $A_\theta = -\frac{e^{-\frac{j\omega r}{c}}}{r}\sin\theta$, $A_\phi = 0$

Thus $\nabla\times\underline{A} = \frac{C}{r^2\sin\theta}\Big\{\hat{\underline{r}}.\left[0-0\right] + r\hat{\underline{\theta}}\left[0-0\right]$

$$+ r\sin\theta\,\hat{\underline{\phi}}\left[\frac{\partial}{\partial r}\left(r.-\frac{e^{-\frac{j\omega r}{c}}}{r}\sin\theta\right) - \frac{\partial}{\partial\theta}\left(\frac{e^{-\frac{j\omega r}{c}}}{r}\cos\theta\right)\right]\Big\}$$

3 marks for correctly establishing that $\hat{\underline{r}}$ and $\hat{\underline{\theta}}$ components = 0 and there is only a $\hat{\phi}$ component

$$= \frac{C.r\sin\theta}{r^2\sin\theta}\left[\frac{j\omega}{c}\sin\theta + \frac{\sin\theta}{r}\right] e^{-\frac{j\omega r}{c}}\,\hat{\underline{\phi}}$$

$$\underline{B} = \nabla\times\underline{A} = \frac{\mu_0 I}{4\pi} e^{j\omega(t-\frac{r}{c})}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]\sin\theta\, dl\,\hat{\underline{\phi}}$$

$\left[d\underline{l}\times\hat{\underline{r}} = dl.\sin\theta.\hat{\underline{\phi}}\right]$ (1 mark) $= \frac{\mu_0 I}{4\pi} e^{j\omega(t-\frac{r}{c})}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right] d\underline{l}\times\hat{\underline{r}}$ 3 marks

as given in lectures