

EMQ MC4 - Answers

MC4.1

The plasma conductivity is $\sigma = -\frac{iNq^2}{\omega m}$

$$|\sigma| = \frac{Nq^2}{\omega m} = \frac{7 \times 10^{10} \times (1.6 \times 10^{-19})^2}{2\pi \times 3 \times 10^6 \times 9.11 \times 10^{-31}} = 1.0436 \times 10^{-4} \Omega^{-1} \text{m}^{-1}$$

or $1.0 \times 10^{-4} \Omega^{-1} \text{m}^{-1}$ 2 sf's
answer b)

MC4.2

The plasma frequency is $\omega_p = \left(\frac{Nq^2}{m\epsilon_0}\right)^{\frac{1}{2}}$

[Use of $v_p \approx 9N^{\frac{1}{2}}$
also gives same
answer to 2 sf's]

$$= \left[\frac{7 \times 10^{10} \times (1.6 \times 10^{-19})^2}{9.11 \times 10^{-31} \times 8.85 \times 10^{-12}}\right]^{\frac{1}{2}} = 1.4909 \times 10^7 \frac{\text{rad}}{\text{s}}$$

or $\omega_p = 2.4 \text{ MHz}$ 2 sf's answer b)

MC4.3

The phase velocity is $v_{ph} = \frac{c}{\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{\frac{1}{2}}}$

$$= \frac{3 \times 10^8}{\left(1 - \left(\frac{1.4909 \times 10^7}{2\pi \times 3 \times 10^6}\right)^2\right)^{\frac{1}{2}}} = 4.9 \times 10^8 \text{ m s}^{-1}$$

answer d)

MC4.4

The refractive index is $n = \left[1 - \frac{\omega_p^2}{\omega^2}\right]^{\frac{1}{2}}$

$$= \left[1 - \left(\frac{1.4909 \times 10^7}{2\pi \times 2.6 \times 10^6}\right)^2\right]^{\frac{1}{2}} = 0.41$$

2 sf's
answer c)

MC4.5

In this case $\omega < \omega_p$ ($v < v_p$) so \underline{B} decays according to $\underline{B} = \underline{B}_0 e^{-|k|z}$

$$|k| = \frac{\omega}{c} \left[\frac{\omega_p^2}{\omega^2} - 1\right]^{\frac{1}{2}} = \frac{2\pi \times 1.8 \times 10^6}{3 \times 10^8} \left[\left(\frac{1.4909 \times 10^7}{2\pi \times 1.8 \times 10^6}\right)^2 - 1\right]^{\frac{1}{2}}$$

$= 0.03238 \text{ m}^{-1}$

$$\underline{B} \rightarrow \underline{B}_0 e^{-0.03238 \times 150} \Rightarrow \underline{B} \text{ decays by factor of } e^{+4.857} = 130$$

2 sf's
answer a)