

MC3 Answers

MC3.1 $\sin \theta_{IC} = \frac{n_2}{n_1} = \frac{1.4}{2.7} \therefore \theta_{IC} = 31.2^\circ$ answer c) (3 s.f.s)

MC3.2 Decay length, $d = \frac{1}{k_T \sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_I - 1}}$

$$n = \frac{ck}{\omega}, k_T = \frac{\omega n_2}{c} = \frac{c}{\omega n_2 \sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_I - 1}}$$

$$\sin \theta_{IC} = \frac{n_2}{n_1} = \frac{1.3}{1.8} = \frac{3 \times 10^8}{2\pi \times 10^{13} \times 1.3 \times \sqrt{\frac{1.8^2}{1.3^2} \sin^2 49^\circ - 1}}$$

$$\Rightarrow \theta_{IC} = 46.2^\circ = 1.21 \times 10^{-5} \text{ m} \quad (2 \text{ s.f.s})$$

In $0.5 \mu\text{m}$ E will decay by factor of $e^{+\frac{1 \times 10^{-5}}{1.21 \times 10^{-5}}} = 2.3$ answer b)

MC3.3 We must use the appropriate Fresnel eqn.

$$\frac{E_{OR}}{E_{OI}} = \frac{\left(\frac{n_1}{n_2}\right) \cos \theta_I - \cos \theta_T}{\left(\frac{n_1}{n_2}\right) \cos \theta_I + \cos \theta_T} \quad \text{Also, } \frac{n_1}{n_2} = \frac{\sin \theta_T}{\sin \theta_I} \Rightarrow \sin \theta_T = \frac{n_1 \sin \theta_I}{n_2}$$

In this case $\sin \theta_T = \frac{1.2}{2.1} \times \sin 40^\circ = 0.367 \therefore \theta_T = 21.55^\circ, \cos \theta_T = 0.93$

$$\frac{E_{OR}}{E_{OI}} = \frac{\frac{1.2}{2.1} \times 0.76604 - 0.93}{\frac{1.2}{2.1} \times 0.76604 + 0.93} = \frac{-0.492}{1.3677} = -0.36 \text{ answer a) (to 2 s.f.s)}$$

MC3.4 This time use $\frac{E_{OT}}{E_{OI}} = \frac{2 \frac{n_1}{n_2} \cos \theta_I}{\frac{n_1}{n_2} \cos \theta_T + \cos \theta_I}$ ($\theta_{IC} = 28.9^\circ$)

(Snell) $\sin \theta_T = \frac{n_1}{n_2} \sin \theta_I = \frac{2.9}{1.4} \sin 23^\circ = 0.8094, \theta_T = 54.03^\circ$
 $\cos \theta_T = 0.5873$

$$\frac{E_{OT}}{E_{OI}} = \frac{2 \times \frac{2.9}{1.4} \times 0.9205}{\frac{2.9}{1.4} \times 0.5873 + 0.9205} = \frac{3.8135}{2.13705} = 1.78 \text{ (1.8 to 2 s.f.s) answer a)}$$

MC3.5 In this case the expression for the reflection coefficient is

$$R_p = \left[\frac{-\cos \theta_I + \frac{n_1}{n_2} \cos \theta_T}{\cos \theta_I + \frac{n_1}{n_2} \cos \theta_T} \right]^2 = \left[\frac{-0.9205 + \frac{2.9}{1.4} \times 0.5873}{0.9205 + \frac{2.9}{1.4} \times 0.5873} \right]^2 = 0.019 \text{ (to 2 s.f.s) answer d)}$$