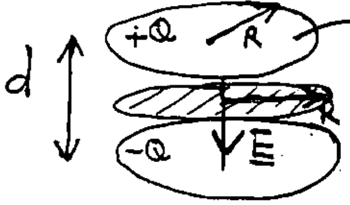
EMQ6 – Answer



area, $A = \pi R^2$

1 mark

a) Work required to raise voltage across capacitor from $0 \rightarrow V$ (= energy stored) is

$$W = \frac{CV^2}{2} = \frac{\epsilon_0 A}{d} \cdot \frac{(Ed)^2}{2} = \tfrac{1}{2}\epsilon_0 E^2 \cdot (\underbrace{Ad}_{\text{volume}})$$

or simply Energy stored = W = energy density $(\tfrac{1}{2}\epsilon_0 E^2)$ x volume (Ad)

1 mark

b) $W = \tfrac{1}{2}\epsilon_0 E^2 (Ad) \quad \therefore \quad \dfrac{\partial W}{\partial t} = \epsilon_0 E \dfrac{\partial E}{\partial t} \cdot (\underbrace{Ad}_{\text{vol}})$

4 marks

c) $$\int (\nabla \times \underline{B}) \cdot d\underline{a} = \mu_0 \int \underbrace{\underline{J} \cdot d\underline{a}}_{=0} + \mu_0 \epsilon_0 \int \frac{\partial \underline{E}}{\partial t} \cdot d\underline{a}$$

[Integration over shaded area shown above]

Stokes Theorem

no current density between plates

$$\oint_{\text{edge of surface}} \underline{B} \cdot d\underline{l} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \cdot \pi R^2$$

[$\underline{E}$ and $d\underline{a}$ parallel
$\underline{E}$, $\dfrac{\partial \underline{E}}{\partial t}$ uniform across area, total area $= \pi R^2$]

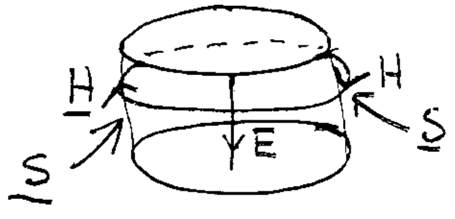
Due to circular symmetry

$$B \cdot 2\pi R = \mu_0 H \cdot 2\pi R = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \cdot \pi R^2$$

$$\therefore \quad H = \frac{\epsilon_0 R}{2} \frac{\partial E}{\partial t} \quad \text{as required.}$$

d)

4 marks



[or equivalent diagram with directions of $\underline{E}$, $\underline{H}$ reversed – $\underline{E}$, $\underline{H}$ perpendicular at all pts.]

$\underline{S}$ is directed radially inwards at all pts. on side of "pill box".

$$\int_{\substack{\text{area enclosing}\\ \text{pill box}}} \underline{S} \cdot d\underline{a} = \int (\underline{E} \times \underline{H}) \cdot d\underline{a} = E \cdot \frac{\epsilon_0 R}{\cancel{2}} \frac{\partial E}{\partial t} \cdot \cancel{2}\pi R d$$

$$= \epsilon_0 E \frac{\partial E}{\partial t} \cdot (\underbrace{d \cdot \pi R^2}_{\text{vol}})$$

no contribution from top and bottom surfaces where $\underline{S}$, $d\underline{a}$ perpendicular.

= same result as b)

[$\Longrightarrow$ energy flowing inwards/time = increase in energy/time within capacitor]