EMQ5 Answer

Background

For simplicity we take $E \propto e^{i(\omega t - kz)}$, $k = k_r - ik_i$.

$\rightarrow k^2 = \omega^2\mu\epsilon - i\mu\sigma\omega \rightarrow \omega^2\mu_0\epsilon - i\mu_0\sigma\omega$, non-magnetic

e.g. to be a good conductor we require that:

$\mu_0\sigma\omega \gg \omega^2\mu_0\epsilon$ or $\sigma \gg \epsilon\omega$

σ: $0.8\,\Omega^{-1}m^{-1}$

$\epsilon\omega$: $15.0 \times 8.85 \times 10^{-12} \times 2\pi \times 1.3 \times 10^{9} = 1.084\,\Omega^{-1}m^{-1}$

$\mu_0\sigma\omega$: $4\pi \times 10^{-7} \times 0.8 \times 2\pi \times 1.3 \times 10^{9} = 0.8212 \times 10^{4}\ m^{-2}$

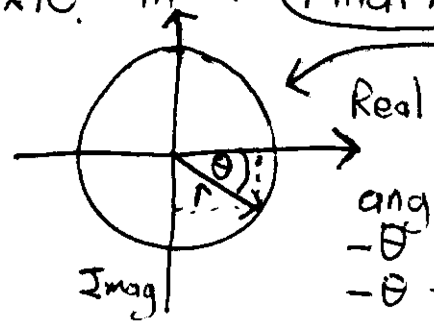
$\omega^2\mu_0\epsilon$: $(2\pi \times 1.3 \times 10^{9})^2 \times 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 15 = 1.113 \times 10^{4}\ m^{-2}$

Clearly neither term dominates, the material is neither a clear good or bad conductor and both terms must be retained. (2 marks for conclusion)

We have that

$$k^2 = (1.113 - 0.8212i) \times 10^{4}\ m^{-2} \quad \text{(1 mark)}$$

$$= re^{-i\theta}$$



2 marks for approach

angle is $-\theta$ or $-\theta + 2\pi$ etc

$$\therefore k = r^{1/2} e^{-\frac{i\theta}{2}}$$

$$\text{or } k = r^{1/2} e^{-i(\frac{\theta}{2} + \pi)}$$

where $r = (1.113^2 + 0.8212^2)^{1/2} \times 10^{4} = 1.383 \times 10^{4}\ m^{-2}$

$r^{1/2} = 1.176 \times 10^{2}\ m^{-1}$

$\theta = \tan^{-1}\left(\frac{0.8212}{1.113}\right) = 36.42^\circ \equiv 0.6357$ radians

$$\therefore k = \pm 1.176 \times 10^{2} (\cos 18.21^\circ - i \sin 18.21^\circ)$$

$$= \pm(1.117 - 0.3675i) \times 10^{2}\ m^{-1} \quad \text{(2 marks)}$$

We take the physically meaningfull solution in which $E \propto e^{-k_i z}$

$\Longrightarrow \propto e^{-36.75z}$

We require that $e^{-36.75z} = 0.1$, $z = \frac{-\ln 0.1}{36.75}$

$\Longrightarrow z \approx \underline{\underline{0.063\ m}}$ or 6.3 cm

(3 marks)