EMQ4 Answer

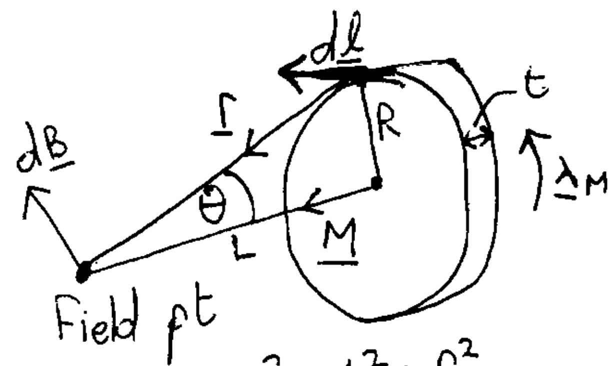
The volume current density, $\underline{J}_M = \nabla \times \underline{M} = 0$ because the disk is uniformly magnetised.

3 marks

The surface current density $\underline{\lambda}_M = \underline{M} \times \hat{\underline{n}}$

$\underline{\lambda}_M = 0$ on front and back surface of disk because $\underline{M}$ and $\hat{\underline{n}}$ are parallel. $\lambda_M = M\sin\theta = 0$

On rim of disk $\underline{M}$, $\hat{\underline{n}}$ are perpendicular and the magnitude of surface current density $\lambda_M = M$



$r^2 = L^2 + R^2$

Total current (I) flowing around rim is just surface current density $\times t$

$\therefore \; I = Mt$ (1)

Because $L \gg t$ we can ignore the detailed surface distribution of current across the width of the rim and just treat it as a current loop with the integral over $d\underline{\ell}$ around the loop. Because the non-axial contribution to $\underline{B}$ from opposite sides of the loop cancel there is only an axial component of $\underline{B}$ along the axis and the contribution from an element $d\underline{\ell}$ is

$$dB = \frac{\mu_0 (\overset{I}{Mt})}{\underbrace{4\pi(L^2+R^2)}_{r^2}} \cdot \underbrace{\sin\theta}_{\text{component along axis} \; = R/(L^2+R^2)^{1/2}} \cdot d\ell$$

$\leftarrow |d\underline{\ell} \times \hat{\underline{r}}| = d\ell$ because $d\underline{\ell}$ and $\hat{\underline{r}}$ are always perpendicular

Thus total axial field is

$$B = \frac{\mu_0 MtR}{4\pi(L^2+R^2)^{3/2}} \underbrace{\int d\ell}_{2\pi R} = \frac{\mu_0 MtR^2}{2(L^2+R^2)^{3/2}}$$

as required

(6)