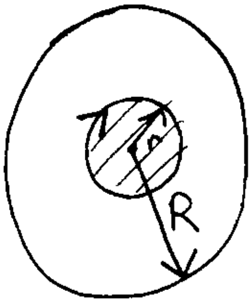
EMQ2 ANSWER

a) I into sheet, say.



We need to integrate $\nabla \times \underline{B} = \mu_0 \underline{J}$ over the shaded area shown. (1 mark)

$$\int (\nabla \times \underline{B}) \cdot d\underline{a} = \mu_0 \int \underline{J} \cdot d\underline{a}$$ ← current through loop of radius r (1 mark)

The uniform current density is just $J = \dfrac{I}{\pi R^2}$ (1 mark)

Stoke's theorem

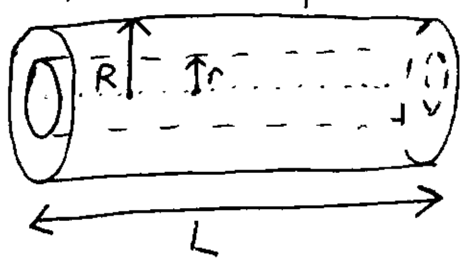
$$\therefore \int (\nabla \times \underline{B}) \cdot d\underline{a} = \oint \underline{B} \cdot d\underline{\ell} = \mu_0 \cdot \frac{I}{\cancel{\pi} R^2} \cdot \cancel{\pi} r^2$$

closed circular loop, radius r

From symmetry $\Rightarrow B \cdot 2\pi r = \dfrac{\mu_0 I r^2}{R^2}$

or $B(r) = \dfrac{\mu_0 I r}{2\pi R^2}$ (2 marks)

b) From symmetry, $\underline{E}$ will be radially directed (in/out depends on sign of ρ).



We need to integrate $\nabla \cdot \underline{E} = \dfrac{\rho}{\epsilon_0}$ over the volume of the inner cylinder (1 mark)

$$\int_{\text{cylinder, radius } r, \text{ length } L} (\nabla \cdot \underline{E}) dv = \int \frac{\rho}{\epsilon_0} dv = \frac{Q}{\epsilon_0}$$ ← charge within cylinder, radius r, length L

The charge within the cylinder is $Q = \pi r^2 L \rho$ (1 mark)

$$\therefore \int (\nabla \cdot \underline{E}) dv = \int \underline{E} \cdot d\underline{a} = \pi r^2 L \rho / \epsilon_0$$

Divergence theorem (1 mark)

(2 marks)

From symmetry $\Rightarrow E . 2\pi r L = \pi r^2 L \rho / \epsilon_0 \Rightarrow E(r) = \dfrac{r\rho}{2\epsilon_0}$