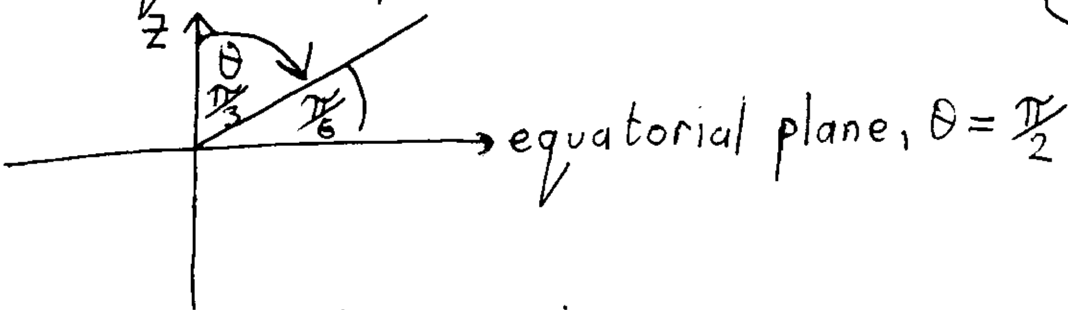
EMQ10 Answer

With $\underline{E}$ as given the maximum value of E (which is the amplitude) is given by $E_0(\theta)$, say, where

$$E_0(\theta) = \frac{c\mu_0 I_0}{2\pi r}\left|\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right|$$ 2 marks

The amplitude at angle θ clearly depends on θ with a minimum amplitude $E_0(\theta)=0$ when $\theta=0$ or $\theta=\pi$ along the dipole axis. In the equatorial plane $E_0(\theta=\frac{\pi}{2})$ has its peak amplitude with $E_0(\frac{\pi}{2}) = \frac{c\mu_0 I_0}{2\pi r}$.

We are interested in the situation shown, $\frac{\pi}{6}$ above the equatorial plane when $\theta = \frac{\pi}{3}$ ← 1 mark



We can find I_0 via

$$I_0 = \frac{2\pi r E_0(\frac{\pi}{3})}{c\mu_0} \cdot \frac{\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{2}\cos\frac{\pi}{3})}$$

We have: $r = 1.5\times10^4$ m, $E_0(\frac{\pi}{3}) = 1.5\times10^{-2}\,Vm^{-1}$, as given in the question 4 marks

$$\therefore I_0 = \frac{2\pi\times1.5\times10^4\times1.5\times10^{-2}}{3\times10^8\times4\pi\times10^{-7}}\times\sqrt{\frac{3}{2}} = \underline{\underline{4.59\text{ amps}}}$$

Total time-averaged power radiated by a half-wave dipole is

$P = \frac{I_0^2 R_r}{2}$ where the radiation resistance $R_r = 73\Omega$ (lectures)

In this case we have $P = \frac{4.59^2\times73}{2} = \underline{\underline{770\text{ watts}}}$

2 marks