Some example examination questions and answers

Q1 Write down the expression for the Poynting vector and explain its significance. Given that the electric and magnetic fields of an electromagnetic wave are represented in phasor form, what is the expression for the time-averaged Poynting vector $\underline{S}_{av}$? [4 marks]

An electromagnetic wave travelling in free space with magnetic induction and electric fields

$$\underline{B} = \underline{B}_o e^{i(\omega t - \underline{k}\cdot\underline{r})}$$
$$\underline{E} = \underline{E}_o e^{i(\omega t - \underline{k}\cdot\underline{r})}$$

where ω, $\underline{k}$, $\underline{B}_o$ and $\underline{E}_o$ are constants, has

$$\underline{S}_{av} = 4.6\times10^{-3}\begin{pmatrix}1\\1\\0\end{pmatrix}\text{W m}^{-2}$$

If $\underline{B}_o$ is in the $\begin{pmatrix}-1\\+1\\+2\end{pmatrix}$ direction what are $\underline{E}_o$ and $|\underline{B}_o|$? If $\omega = 10^{15}$ radians s^{-1} what is $\underline{k}$? [6 marks]

ANSWER

Q2 Write down the form of Maxwell's equations in free space and show that they lead to the following wave equation for the electric field:

$$\nabla^2\underline{E} = \mu_o\varepsilon_o\frac{\partial^2\underline{E}}{\partial t^2}$$

[4 marks]

It is known that a solution of the above equation within the interior of a hollow square metallic tube with cross-sectional width a is

$$E_x = E_o\sin\left(\frac{\pi y}{a}\right)e^{i(\omega t - kz)}$$
$$E_y = E_z = 0$$

where the z-axis is parallel to the axis of the tube and ω, k and E_o are constants. Prove that if a = 0.05 m a wave can only freely propagate along the tube at frequencies above 3 GHz. [4 marks]

By considering the symmetry of the tube write down another solution of the above wave equation propagating in the same direction. [2 marks]

$$[\ \nabla \times \nabla \times \underline{C} = \nabla\,(\nabla .\ \underline{C}\ ) - \ \nabla^2 \underline{C}\ ]$$

ANSWER

Short questions

Q3 a) An EM wave is propagating in the $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$ direction. Suggest possible directions for $\underline{E}$ and $\underline{B}$. [2 marks]

b) A plane electromagnetic wave in free space has a *B*-field amplitude of $B_o = 6.67 \times 10^{-9}$ T. What is the magnitude of the time-averaged Poynting vector for this EM wave? [2 marks]

c) An object carrying a peak current of 2.5 A radiates a total EM power of 500 W. What is the value of the radiation resistance? [2 marks]

d) A material has uniform magnetisation given by $\underline{M} = 1.0 \times 10^{-4} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \mathrm{Am}^{-1}$. Given that a normal to the surface of the material is in the $\begin{pmatrix} -1 \\ +1 \\ +2 \end{pmatrix}$ direction what is the value of the surface magnetisation current density on this surface? [2 marks]

e) A region of the interstellar medium has an associated plasma frequency of 1.5 kHz. What is the electron concentration in this region? [2 marks]

f) At a frequency of 8.0×10^8 Hz what is the skin depth for a plane EM wave in copper given that the conductivity is $\sigma = 6 \times 10^7\ \Omega^{-1}\mathrm{m}^{-1}$? [2 marks]

g) Explain how the vector potential $\underline{A}$ is related to the magnetic induction $\underline{B}$. In the time-dependent case show how the electric field $\underline{E}$ is related to both $\underline{A}$ and the scalar potential *V*. [2 marks]

ANSWERS TO SHORT QUESTIONS