Level 2 Electromagnetism Example Questions 2002/3 – Sheet 6

Q24 The far-field result obtained in lectures for the Hertzian dipole in free space is

$$\underline{B}(\underline{r},t) \approx j\frac{\mu_o I_o}{2\lambda r} dl \sin\theta \; e^{j\omega(t-r/c)} \underline{\hat{\phi}} = B_\phi \underline{\hat{\phi}}$$

which has a ϕ component only.

By using this result together with the appropriate Maxwell equation prove that the corresponding form of $\underline{E}$, as given in lectures, is

$$\underline{E}(\underline{r},t) \approx j\frac{c\mu_o I_o}{2\lambda r} dl \sin\theta \; e^{j\omega(t-r/c)} \underline{\hat{\theta}}$$

which has a θ component only.

In the spherical polar coordinate system

$$\nabla\times\underline{C} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \underline{\hat{r}} & r\underline{\hat{\theta}} & r\sin\theta\,\underline{\hat{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ C_r & rC_\theta & r\sin\theta C_\phi \end{vmatrix}$$

Q25 Imagine that an object radiates electromagnetic waves with a power distribution given by

$$\underline{S}_{av}(r,\theta) = \frac{CI_o^2}{r^2}\sin\theta\,\underline{\hat{r}}$$

where $C = 5\ \Omega$. What is the value of the radiation resistance for this object? What is the beam width? What is the value of the directivity?

Q26 ……. And now, a rather different question.

A superconductor is a material which offers no dc resistance and satisfies the following relationship connecting current density and magnetic field under steady-state (no variation with time) conditions:

$$\nabla\times\underline{J} = -\frac{nq^2}{M}\underline{B}$$

Show that this leads to the following equation for $\underline{B}$:

$$\nabla^2\underline{B} = \frac{\mu_o nq^2}{M}\underline{B}$$

Show that $B_x = 0,\ B_y = 0,\ B_z(x) = B_o e^{-x/\lambda}$ is a consistent solution for $\underline{B}$ and find an expression for λ. B_o is a constant equal to the strength of the *B* - field at the surface of the superconductor. (The magnetic field within the superconductor thus decays exponentially from the surface value.)

Obtain an expression for $\underline{J}$ involving B_o and λ.

Given the values below find the numerical value of the "penetration depth", λ.

M = 2 x electron mass, q = 2 x electron charge, n (concentration) = 5 x 10^{28} m^{-3}