**Level 2 Electromagnetism Example Questions – Sheet 5 2002/3**

Q19 When electromagnetic radiation impinges at normal incidence on the planar interface between a particular combination of two non-magnetic, non-conducting dielectric media it is found that the reflection and transmission coefficients are identical. Derive an expression from which the possible values for the ratio $n_1/n_2$ (where $n_1$ and $n_2$ are the refractive indices of the two media) can be obtained. If one of the media has a relative permittivity of 9.0 what are the possible combinations of refractive indices for the 2 media?

Q20 In a high density plasma, such as a metal, collisions between electrons cannot be ignored and the average response of electrons to an applied *E*-field satisfies:

$$m\frac{d\underline{v}}{dt} = -q\underline{E} - \frac{m\underline{v}}{\tau}$$

where $\tau$ is the mean time between collisions. Show that this leads to the expression

$$\sigma = \frac{Nq^2}{m(i\omega + 1/\tau)}$$

for the plasma conductivity, $\sigma$. Given that the electron concentration is $N = 8.5\times10^{28}$ m$^{-3}$ use the low frequency value of $\sigma$ for copper ($6\times10^7$ $\Omega^{-1}$m$^{-1}$) to obtain an estimate for the value of $\tau$. At what frequency in Hz is the magnitude of the conductivity reduced to half its low frequency value?

Q21 Hex C Spanner broadcasts a pirate radio station programme on the medium wave frequency of 1.5 MHz. Show why this signal cannot propagate through the ionosphere. Given that the peak electron concentration in the ionosphere decays according to $N(t) = N_o e^{-t/T}$ after sunset, and $T$ = 2 hours, estimate how long after sunset it will be before his early evening rap extravaganza begins to propagate outwards to a wider, extra-terrestrial audience. [Take $N_o = 10^{11}$ m$^{-3}$.]

Q22 Demonstrate that if $\omega >> \omega_p$ for a plasma then

$$v_g \approx c\left(1 - \frac{\omega_p^2}{2\omega^2}\right)$$

Further, show that $v_{ph} + v_g \approx 2c$.

If a 110 Mhz radio pulse from the Crab Nebula (distance ~ 6 x $10^{19}$ m) arrives 1.5 s later than a 115 MHz pulse (produced at the same time) estimate the plasma frequency in hertz and also the average electron density of the interstellar medium.

Q23 In considering the time dependent form of Maxwell's eqns. show how the choice of Lorentz gauge in which

$$\nabla.\underline{A} = -\mu_o\varepsilon_o\frac{\partial V}{\partial t}$$

leads to the following results:

$$\nabla^2 V = \frac{-\rho}{\varepsilon_o} + \mu_o\varepsilon_o\frac{\partial^2 V}{\partial t^2} \quad \text{and} \quad \nabla^2\underline{A} = -\mu_o\underline{J} + \mu_o\varepsilon_o\frac{\partial^2\underline{A}}{\partial t^2}$$

[Note that the similar form of these two equations means that the solutions for the individual components of $\underline{A}$ must be of the same form as the solution for $V$.]