Level 2 Electromagnetism Example Questions – Sheet 4 2002/3

Q14 A plane electromagnetic wave in free space has an associated electric field amplitude of $E_o = 1.5 \times 10^{-3}$ Vm^{-1}. What is the amplitude of the associated H - field and what is the value of S_{ave} for this wave?

Q15 By considering an imaginary smaller cylinder with radius r concentric with a larger air-filled cylindrical solenoid (within which $\underline{B}$ and hence $\underline{H}$ are spatially uniform but changing with time) prove that:

$$\int_{surface} \underline{S} \cdot d\underline{a} = -\frac{\partial W}{\partial t}$$

where $\underline{S} = \underline{E} \times \underline{H}$ is Poynting's vector, W = $(\mu_o H^2/2) \times$ volume is the energy associated with the presence of the magnetic field within the imaginary cylinder and the surface integral is over the entire surface of the imaginary cylinder.

[Hint: begin by using Maxwell's second equation to show that $E = -(\mu_o r/2)\partial H/\partial t$ tangential to the main surface of the cylinder.]

Q16 A coaxial cable is composed of a long straight metallic wire of radius a surrounded by a concentric cylindrical metallic sheath of inner radius b with air in between. An electromagnetic wave travelling within the cable has electric and magnetic fields given by:

$$\underline{E} = \frac{Ve^{j(\omega t - kz)}}{r\ln(b/a)}\hat{\underline{r}} \quad , \quad \underline{B} = \frac{Ve^{j(\omega t - kz)}}{rc\ln(b/a)}\hat{\underline{\phi}}$$

where $\hat{\underline{r}}$ and $\hat{\underline{\phi}}$ are unit vectors of the cylindrical coordinate system used to describe this situation and z is the direction of propagation, along the length of the cable. V is a real constant.

Sketch the pattern of the $\underline{E}$ and $\underline{B}$ fields at the time $t = 0$ in the plane z = 0 (the plane perpendicular to the axis of the wire).

By integrating over the cross section of the cable in the air-filled region obtain an expression for the time-averaged power flow along the coaxial cable.

Q17 Beginning with the boundary conditions

$$E_{OI} + E_{OR} = E_{OT} \qquad 1)$$
$$H_{OI}\cos\theta_I - H_{OR}\cos\theta_I = H_{OT}\cos\theta_T \qquad 2)$$

with the electric field polarised normal to the plane of incidence, prove that

$$\frac{H_{OR}}{H_{OI}} = \frac{?}{\frac{n_1}{\mu_1}\cos\theta_I + \frac{n_2}{\mu_2}\cos\theta_T} \quad , \quad \frac{H_{OT}}{H_{OI}} = \frac{?}{\frac{n_1}{\mu_1}\cos\theta_I + \frac{n_2}{\mu_2}\cos\theta_T}$$

Q18 Two non-conducting LIH media have identical permittivities but different permeabilities. By making use of a Fresnel equation with E polarised normal to the plane of incidence prove that there is no reflected wave if $\tan\theta_I = n_2/n_1$.

[This is a magnetic material analogue of the standard Brewster situation for dielectric materials.]