Level 2 Electromagnetism Example Questions – Sheet 3 2002/3

Q9 Prove (by using the appropriate Maxwell equation) that for a plane electromagnetic wave in a general LIH medium ($k^2 = \omega^2\mu\varepsilon - i\mu\sigma\omega$) with

$$\underline{B} = \underline{B}_0 e^{j(\omega t - \underline{k}\cdot\underline{r})} = \begin{pmatrix} B_{0x} \\ B_{0y} \\ B_{0z} \end{pmatrix} e^{j(\omega t - k_x x - k_y y - k_z z)}$$

that $\underline{E} = \frac{\omega}{k^2}\underline{B}\times\underline{k}$

Q10 Given the relationship

$$k^2 = (k_r - jk_i)^2 = \omega^2\mu\varepsilon - j\mu\sigma\omega$$

for a linear, isotropic and homogeneous medium, prove that the following general expression can be obtained for the phase velocity:

$$v_{ph} = \frac{\omega}{k_r} = \frac{1}{(\mu\varepsilon)^{1/2}}\left[\frac{2}{1+(1+\frac{\sigma^2}{\omega^2\varepsilon^2})^{1/2}}\right]^{1/2}$$

Demonstrate that the expected results for the phase velocities in poor and good conductors are obtained in the appropriate limiting cases.

[You may also wish to consider how you would obtain an expression for the group velocity.]

Q11 By what factor will the B-field of a 3 MHz EM wave be attenuated in travelling a distance of 2 m through seawater?

Q12 A mobile phone placed against your ear gives rise to a resultant electric field strength E = 25 Vm^{-1} in the air at the surface of your head. Assuming that the transmission frequency corresponds to that of the Orange digital network (1800 MHz) and given that ε_{rel} = 51.6 and σ = 1.6 $\Omega^{-1}m^{-1}$ for brain tissue demonstrate that brain tissue can be considered neither a good nor a particularly poor conductor at this frequency. Bearing in mind this result, and neglecting the presence of your skull which has a relatively low conductivity, make a rough estimate of the consequent electric field, E, 3 cm inside your brain.

What value (incorrect) is obtained if you simply employ the good conductor approximation to evaluate δ and use this result to compute E 3 cm within your brain?

[Note: It is something of an oversimplification to completely neglect the presence of the skull and also to use the standard plane-wave model here but this is sufficiently good in order to obtain a rough estimate of the electric field strength.]

Q13 Beginning with $\underline{E} = \begin{pmatrix} E_o \\ 0 \\ 0 \end{pmatrix} e^{i(\omega t - kz)}$ within a good conductor show that $\underline{E}$ and $\underline{B}$ are out of phase with each other by $\pi/4$.