Level 2 Electromagnetism Example Questions 2002/3 – Sheet 2

Q4 For a standard plane EM wave in free space with

$$\underline{E} = \underline{E}_0 e^{j(\omega t - \underline{k}\cdot\underline{r})} = \begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix} e^{j(\omega t - k_x x - k_y y - k_z z)} \text{ and } \underline{B} = \underline{B}_0 e^{j(\omega t - \underline{k}\cdot\underline{r})} = \begin{pmatrix} B_{0x} \\ B_{0y} \\ B_{0z} \end{pmatrix} e^{j(\omega t - k_x x - k_y y - k_z z)}$$

prove that $\nabla \cdot \underline{E} = 0$ and $\nabla \cdot \underline{B} = 0$ can only be always true provided that $\underline{E}$ and $\underline{B}$ are perpendicular to the direction of propagation given by $\underline{k}$.

Q5 a) Prove (by using the appropriate Maxwell equation) that for a plane electromagnetic wave in free space with $\underline{E}$ as in Q4 that $\underline{B} = \dfrac{\underline{k} \times \underline{E}}{\omega}$.

b) Prove (by using the appropriate Maxwell equation) that for a plane electromagnetic wave in free space with $\underline{B}$ as in Q4 that $\underline{E} = \dfrac{\omega}{k^2}\underline{B} \times \underline{k}$.

Q6 A plane EM wave as in Q4 has $\underline{k}$ in the $\begin{pmatrix} +1 \\ -2 \\ +3 \end{pmatrix}$ direction.

a) If $\underline{E}_o = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$ Vm^{-1} what is $\underline{B}_o$?

b) If $\underline{B}_o = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$ T what is $\underline{E}_o$?

c) If the frequency of the wave is 10^6 Hz what are the corresponding $\underline{k}$ values for the situations a) and b) above?

Q7 When a particular uniform electric field is applied to a sample of diamond the induced polarisation is $P_o = 1.5 \times 10^{-7}$ Cm^{-2}.

a) Calculate the number of carbon atoms per m^3 in diamond given the information below.

b) Calculate the average induced dipole moment per carbon atom.

c) Estimate the average separation between the centres of +ve/-ve charge (ie the carbon nucleus and surrounding electron charge cloud) in the diamond.

d) If $P = P_o \sin(\omega t)$ as a result of an alternating electric field evaluate the peak value of the resultant polarisation current density, J_b, at a frequency of 10^{12} Hz.

Information: The density of diamond is 3500 kgm^{-3}.
1 kmole of carbon has a mass of 12 kg.
Avogadro's constant is 6.0 x 10^{26} $kmol^{-1}$.
The atomic number of carbon is 6.

Q8 The polarisation at the surface of a spherical object of radius *R* centred at the origin is $\underline{P} = C(x\hat{\underline{i}} + y\hat{\underline{j}} + z\hat{\underline{k}})$ where C is a constant. What is the value of the total charge required within the volume of the sphere in order to ensure that it has no net charge?