Level 2 Electromagnetism Example Questions 2002/3 – Sheet 1

Q1 When a voltage $v(t) = V_o\cos(\omega t)$ is applied to a circuit element the resultant current is given by $i(t)=I_o\cos(\omega t + \varphi)$ where φ is a constant. Prove that the time-averaged power dissipation is given by $V_oI_o\cos(\varphi)/2$. Show that the same result is obtained (more easily) by using a phasor representation for the voltage and current.

Q2 A conducting disk (with radius b) is rotated about its axis at angular velocity ω on a non-conducting spindle (with radius a) in the presence of a uniform magnetic field (B) parallel to the axis. By considering the magnetic force on the electrons in the disc consider how this will cause the electrons to distribute themselves within the disc. What will be the consequence of this charge redistribution? On the assumption that the resultant forces on the electrons in the disc are in equilibrium, derive an expression for the open circuit voltage, V_{oc}, developed between the rim and spindle.

If B = 1.5 T and the radii for the spindle and disk are 3 cm and 40 cm respectively what is the value of V_{oc} when the rotation rate is 2000 rpm?

[The above is an example of a simple *homopolar generator* system and can be used as the basis for a direct current source. When used in reverse, the disc is caused to rotate as a result of the application of a direct voltage between the rim and spindle – a *homopolar motor*.]

Q3 This is a reminder of some elementary Level 1 EM results and emphasises the need to consider the *symmetry* of a problem in order to quickly obtain a result.

Find expressions for the electric (*E*) or magnetic (*B*) fields in the following situations:

a) The *E* - field at a distance *r* from an infinite line of charge with linear charge density μ Cm^{-1}.

b) The *E* – field at a distance *h* above an infinite sheet of charge with area charge density σ Cm^{-2}. (You should also be able to easily deduce the expression for *E* between the plates of a capacitor with $\pm\sigma$ Cm^{-2} on the two plates.)

c) The *E* – field at a distance *r* from the centre of a sphere of radius *R* ($r > R$) containing a volume charge density ρ Cm^{-3}.

d) The *B* – field at a distance *r* from an infinite straight wire carrying a current *I*.

e) The *B* – field inside an infinite circular solenoid with *N* turns m^{-1} carrying a current *I*.