Answer to Electromagnetism Example Question 9

Now $\nabla\times\underline{B} = \mu\underline{J} + \mu\varepsilon\frac{\partial\underline{E}}{\partial t}$

$$= \mu\sigma\underline{E} + \mu\varepsilon\frac{\partial\underline{E}}{\partial t}$$

$$= \mu\sigma\underline{E} + i\omega\mu\varepsilon\underline{E}$$

Although $\underline{k}$ is now complex we still have the result that $\nabla\times\underline{B} = -i\underline{k}\times\underline{B}$.
Therefore, $-i\underline{k}\times\underline{B} = (\mu\sigma + i\omega\mu\varepsilon)\underline{E}$

or $$\underline{E} = \frac{-i\underline{k}\times\underline{B}}{(\mu\sigma + i\omega\mu\varepsilon)}$$

Multiplying top and bottom with $-i$ we get

$$\underline{E} = \frac{-\underline{k}\times\underline{B}}{(\omega\mu\varepsilon - i\mu\sigma)}$$

$$= \frac{-\omega\underline{k}\times\underline{B}}{(\omega^2\mu\varepsilon - i\mu\sigma\omega)}$$

$\Rightarrow \underline{E} = \frac{\omega}{k^2}\underline{B}\times\underline{k}$ as required because $k^2 = \omega^2\mu\varepsilon - i\mu\sigma\omega$.

The proof is thus more involved but the final appearance of this result is exactly the same as we had for free space.