Answer to Electromagnetism Example Question 8

The surface charge density is given by the standard expression $\sigma_b = \underline{P} \cdot \hat{\underline{n}}$.

In this case $\underline{P}$ is clearly radially directed and so is $\hat{\underline{n}}$ as it is perpendicular to the surface of a sphere.

The unit vector in the radial direction can be written as

$$\hat{\underline{n}} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} . (x\hat{\underline{i}} + y\hat{\underline{j}} + z\hat{\underline{k}})$$

$$\therefore \sigma_b = \underline{P} \cdot \hat{\underline{n}} = \frac{\mathrm{C}\left(x^2 + y^2 + z^2\right)}{\sqrt{x^2 + y^2 + z^2}} = \frac{\mathrm{C}R^2}{R} = CR$$ at the surface of the sphere.

Therefore the total charge appearing on the surface is just

$$\sigma_b \times \text{ area} = \mathrm{C}R \times 4\pi R^2 = 4\pi \mathrm{C}R^3$$

In order to ensure overall charge neutrality it is then necessary to have a charge of $-4\pi \mathrm{C}R^3$ within the interior of the sphere.