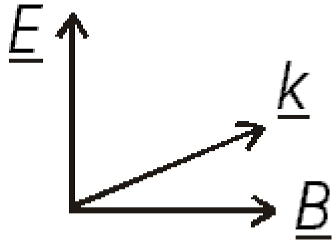
Answer to Electromagnetism Example Question 6



a) $\underline{B}_o$ is in the *direction* of $\underline{k}\times\underline{E}$ and hence in the direction of $\begin{pmatrix}+1\\-2\\+3\end{pmatrix}\times\begin{pmatrix}-2\\-1\\0\end{pmatrix}=\begin{pmatrix}+3\\-6\\-5\end{pmatrix}$.

The *unit* vector in this direction is $\dfrac{1}{\sqrt{3^2+6^2+5^2}}\begin{pmatrix}+3\\-6\\-5\end{pmatrix}=\dfrac{1}{\sqrt{70}}\begin{pmatrix}+3\\-6\\-5\end{pmatrix}$.

We also know that $B_o = E_o/\mathrm{c}$ in free space and in this case

$E_o=\sqrt{2^2+1^2+0^2}=\sqrt{5}$ Vm^{-1} .

$$\therefore \underline{B}_o=\frac{1}{\mathrm{c}}\cdot\sqrt{\frac{5}{70}}\begin{pmatrix}+3\\-6\\-5\end{pmatrix}=\frac{0.267}{\mathrm{c}}\begin{pmatrix}+3\\-6\\-5\end{pmatrix}=8.9\times10^{-10}\begin{pmatrix}+3\\-6\\-5\end{pmatrix}\text{T.}$$

b) $\underline{E}_o$ is in the *direction* of $\underline{B}\times\underline{k}$ and hence in the direction of $\begin{pmatrix}-2\\-1\\0\end{pmatrix}\times\begin{pmatrix}+1\\-2\\+3\end{pmatrix}=\begin{pmatrix}-3\\+6\\+5\end{pmatrix}$.

The *unit* vector in this direction is $\dfrac{1}{\sqrt{3^2+6^2+5^2}}\begin{pmatrix}-3\\+6\\+5\end{pmatrix}=\dfrac{1}{\sqrt{70}}\begin{pmatrix}-3\\+6\\+5\end{pmatrix}$.

We also know that $E_o=\mathrm{c}B_o$ in free space and that $B_o=\sqrt{5}$ T.

$$\therefore \underline{E}_o=\mathrm{c}\sqrt{\frac{5}{70}}\begin{pmatrix}-3\\+6\\+5\end{pmatrix}=0.267\mathrm{c}\begin{pmatrix}-3\\+6\\+5\end{pmatrix}=8.01\times10^{7}\begin{pmatrix}-3\\+6\\+5\end{pmatrix}\text{Vm}^{-1}.$$

c) In free space we have that the phase velocity $=\mathrm{c}=\dfrac{\omega}{k}$, $k=\dfrac{\omega}{\mathrm{c}}$.

In this case $\omega=2\pi.10^6$ and so $k=\dfrac{2\pi.10^6}{3\times10^8}=0.0209\,\text{m}^{-1}$.

Normalising our $\underline{k}$ direction vector we obtain

$$\underline{k}=\frac{0.0209}{\sqrt{14}}\begin{pmatrix}+1\\-2\\+3\end{pmatrix}=5.6\times10^{-3}\begin{pmatrix}+1\\-2\\+3\end{pmatrix}\text{m}^{-1}.$$