**Answer to Electromagnetism Example Question 5**

a) Knowing $\underline{E}$ the appropriate Maxwell equation to obtain $\underline{B}$ is $\nabla \times \underline{E} = -\dfrac{\partial \underline{B}}{\partial t} = -j\omega \underline{B}$.

$$\text{Now } \nabla \times \underline{E} = \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} E_{ox} \\ E_{oy} \\ E_{oz} \end{pmatrix} e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$= \begin{pmatrix} -jk_y E_{oz} + jk_z E_{oy} \\ -jk_z E_{ox} + jk_x E_{oz} \\ -jk_x E_{oy} + jk_y E_{ox} \end{pmatrix} e^{j(\omega t - \underline{k}\cdot\underline{r})}$$

$$= -j\underline{k} \times \underline{E}_o e^{j(\omega t - \underline{k}\cdot\underline{r})} = -j\underline{k} \times \underline{E} = -j\omega \underline{B}$$

$\therefore \underline{B} = \dfrac{\underline{k} \times \underline{E}}{\omega}$ as required.

This is a more general proof of the specific result obtained in lectures.

b) Knowing $\underline{B}$ the appropriate Maxwell equation to obtain $\underline{E}$ is

$$\nabla \times \underline{B} = \mu_o \varepsilon_o \frac{\partial \underline{E}}{\partial t} = j\omega \mu_o \varepsilon_o \underline{E} = \frac{j\omega}{c^2} \underline{E} = j\omega . \frac{k^2}{\omega^2} \underline{E} = j\frac{k^2}{\omega} \underline{E}$$

$$\text{Now } \nabla \times \underline{B} = \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} B_{ox} \\ B_{oy} \\ B_{oz} \end{pmatrix} e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$= \begin{pmatrix} -jk_y B_{oz} + jk_z B_{oy} \\ -jk_z B_{ox} + jk_x B_{oz} \\ -jk_x B_{oy} + jk_y B_{ox} \end{pmatrix} e^{j(\omega t - \underline{k}\cdot\underline{r})}$$

$$= -j\underline{k} \times \underline{B}_o e^{j(\omega t - \underline{k}\cdot\underline{r})} = -j\underline{k} \times \underline{B}$$

$\therefore -j\underline{k} \times \underline{B} = j\dfrac{k^2}{\omega} \underline{E}$ or $\underline{E} = \dfrac{\omega}{k^2} \underline{B} \times \underline{k}$ as required.

$[\underline{k} \times \underline{B} = -\underline{B} \times \underline{k}]$