Answer to Electromagnetism Example Question 25

This is NOT meant to correspond to the result for any actual object – it's just a simple mathematical form to easily enable the calculation of R_r ,W and D.

To find the total power radiated, P, we must integrate the time-averaged Poynting vector over a spherical surface surrounding and centred on the object:

$$P = \int \underline{S}_{av} \cdot d\underline{a} = CI_o^2 \int_0^{2\pi} d\phi \int_0^{\pi} \frac{r^2 \sin^2\theta}{r^2} d\theta$$

where, in spherical polars, $d\underline{a} = r^2 \sin\theta \; d\theta \; d\phi \; \hat{\underline{r}}$.

Thus we obtain

$$P = CI_o^2 \times 2\pi \times \int_0^{\pi} \sin^2\theta \; d\theta = CI_o^2 \times 2\pi \times \frac{\pi}{2} = CI_o^2 \pi^2$$

[The simple $sin^2\theta$ type integral was previously performed in doing Example Question 1.]

The radiation resistance is defined via the relationship:

$$P = \frac{1}{2} I_o^2 R_r$$

which, in this case, means that $P = \frac{1}{2} I_o^2 R_r = CI_o^2 \pi^2$ or $R_r = 2C\pi^2 = \underline{98.7}$ Ω with C = 5 Ω as given in the question.

S_{av} has a maximum in the equatorial plane where $\theta = \pi/2$ and falls to half this peak value when $\theta = \pi/6$, $5\pi/6$ when $\sin\theta = 1/2$. This means that the beam width, W = <u>2π/3</u> or <u>120 degrees</u>.

The directivity is given by $D = \frac{\text{peak power/area}}{\text{average power/area}}$.

The peak power/area is obtained in the equatorial plane when $\theta = \pi/2$, $\sin\theta = 1$ and

$$S_{av} = \frac{5I_o^2}{r^2} .$$

The average power/area is just $\frac{\frac{1}{2} I_o^2 R_r}{4\pi r^2}$.

Thus $D = \frac{2 \times 5 I_o^2 \times 4\pi r^2}{r^2 I_o^2 R_r} = \frac{40\pi}{R_r}$ and with R_r = 98.7 Ω this gives <u>D = 1.27</u> somewhat smaller than the standard result for a Hertzian half-wave dipole of 1.5 or the half-wave dipole result of 1.64 as would be expected given the $\sin\theta$ dependence.