Answer to EM Example Q24

The appropriate Maxwell eqn. is

$$\nabla \times \underline{B} = \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t} = j\omega \mu_0 \epsilon_0 \underline{E} = \frac{j\omega}{c^2} \underline{E}$$

[$\mu_0 \underline{J} = 0$ because we are in free space, well away from the dipole]

As $\nabla\times$ operates on positional coordinates we can take $j\frac{\mu_0 I_0}{2\lambda} dl\, e^{j\omega t} = K$, constant.

Thus $\underline{B} = K \sin\theta \frac{e^{-j\omega r/c}}{r} \hat{\underline{\phi}} = B_\phi \hat{\underline{\phi}}$

We then have that

$$\nabla \times \underline{B} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{\underline{r}} & r\hat{\underline{\theta}} & r\sin\theta \hat{\underline{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r.\sin\theta B_\phi \end{vmatrix}$$

$\uparrow B_r = 0$ $\nwarrow B_\theta = 0$

$\nabla \times \underline{B}$ clearly has <u>no</u> $\hat{\underline{\phi}}$ component

The radial $(\hat{\underline{r}})$ component of $\nabla \times \underline{B}$ is

$$\frac{K}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}(r\sin\theta . B_\phi) - \frac{\partial}{\partial\phi}(0)\right] = \frac{K}{r^2\sin\theta} . \cancel{r} \frac{e^{-j\omega r/c}}{\cancel{r}} . 2\sin\theta.\cos\theta$$

This term is $\propto 1/r^2$ but we have already discarded terms $\propto 1/r^2$ in the far-field limit as being insignificant cf $1/r$ terms (see next) so discard this one also $\Longrightarrow$ <u>no significant $\hat{\underline{r}}$ component</u>, ≈ 0.

The $\hat{\underline{\theta}}$ component of $\nabla \times \underline{B}$ is

$$\frac{K}{r^2 \sin\theta} . r . \left[\frac{\partial}{\partial\phi}(0) - \frac{\partial}{\partial r}\left(\cancel{r\sin\theta} . \sin\theta \frac{e^{-j\omega r/c}}{\cancel{r}}\right)\right] = K\sin\theta . \frac{j\omega}{cr} e^{-j\omega r/c}$$

$$\therefore \frac{\cancel{j}K\cancel{\omega}}{\cancel{c}r}\sin\theta\, e^{-j\omega r/c}\,\hat{\underline{\theta}} = \frac{\cancel{j\omega}}{c^{\cancel{2}}}\underline{E} \quad \text{and hence}$$

$$\underline{E} = \frac{jc\mu_0}{2\lambda r} I_0 dl \sin\theta\, e^{j\omega(t - r/c)} \hat{\underline{\theta}} \quad \underline{\text{as required}}$$