Answer to EM Example Q21

Before sunset we take the electron conc^{n} to be $N = 10^{11}\ m^{-3}$ so

$$\nu_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\left(\frac{Nq^2}{m\epsilon_0}\right)^{1/2} = \underline{2.84\ MHz} \quad 3\ \text{s.f's}$$

This is above Hex C's broadcast frequency so the signal cannot pass through the ionosphere.

We need to wait until the electron concentration falls to a value given by

$$N(t) = \frac{m\epsilon_0 \nu_p^2 (2\pi)^2}{q^2} = \frac{9.1\times10^{-31} \times 8.85\times10^{-12} \times (2\pi)^2 \nu_p^2}{(1.6\times10^{-19})^2} = \underline{2.79\times10^{10}\ m^{-3}}$$

[Corresponding to Hex C's broadcast frequency $= \nu_p$.] $= 1.5\times10^{6}\ Hz$

We require that

$$N(t) = N_0 e^{-t/\tau}$$

ie $2.79\times10^{10} = 10^{11} e^{-t/2}$

$$\therefore\ t = -2\ln\left(\frac{2.79\times10^{10}}{10^{11}}\right)$$

$$\Longrightarrow t = \underline{2.55\ hrs}$$

$$\simeq \underline{9200\ seconds}$$

[Precise values depend on precision of calc., use of $\omega_p = \left(\frac{Nq^2}{m\epsilon_0}\right)^{1/2}$ or less precise $\nu_p \approx 9N^{1/2}$.]