

Answer to EM Example Q20

Assume that $\left. \begin{matrix} \underline{E} \\ \underline{v} \end{matrix} \right\} = \left. \begin{matrix} E_0 \\ v_0 \end{matrix} \right\} e^{i(\omega t - \underline{k} \cdot \underline{r})}$

Insertion in given eqn. leads to

$$m \cdot i\omega \underline{v} = -q \underline{E} - \frac{m \underline{v}}{\tau}$$

$$\underline{v} = \frac{-q \underline{E}}{m(i\omega + \frac{1}{\tau})}$$

But $\underline{J} = -Nq \underline{v} = \sigma \underline{E}$ and therefore

plasma conductivity $\sigma = [-Nq] \cdot \left[\frac{-q}{m(i\omega + \frac{1}{\tau})} \right]$

[required expression] $\sigma = \frac{Nq^2}{m(i\omega + \frac{1}{\tau})}$

At low frequencies (constant σ) $\sigma \approx \frac{Nq^2 \tau}{m}$

$$\begin{aligned} \Rightarrow \tau &= \frac{m \sigma}{Nq^2} \\ &= \frac{9.1 \times 10^{-31} \times 6 \times 10^7}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2} \\ \tau &= 2.5 \times 10^{-14} \text{ s} \end{aligned}$$

$|\sigma|$ reduced by factor of 2 when $\omega^2 + \frac{1}{\tau^2} = \frac{4}{\tau^2}$

$$\Rightarrow \omega = \frac{1}{\tau} (4 - 1)^{\frac{1}{2}} = 6.9 \times 10^3$$

$$\underline{\nu} = 1.01 \times 10^{13} \text{ Hz}$$