Answer to Electromagnetism Example Question 1

The instantaneous power dissipation is

$$v(t).i(t) = V_o I_o \cos(\omega t)\cos(\omega t + \varphi)$$

with

$$\omega = 2\pi f \quad , \text{ period } \; T = \frac{1}{f} = \frac{2\pi}{\omega}$$

The time-averaged power dissipation is:

$$\begin{aligned} P &= V_o I_o . \frac{1}{T}\int_0^T \cos(\omega t)\cos(\omega t + \varphi)dt \\ &= V_o I_o . \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \cos(\omega t)\left[\cos(\omega t)\cos(\varphi) - \sin(\omega t)\sin(\varphi)\right]dt \\ &= \frac{V_o I_o \omega}{2\pi}\left[\cos(\varphi)\int_0^{2\pi/\omega}\cos^2(\omega t)dt - \sin\varphi\int_0^{2\pi/\omega}\cos(\omega t)\sin(\omega t)dt\right] \end{aligned}$$

The second of these integrals = 0, and the first can also be written as:

$$\frac{V_o I_o \omega}{2\pi}\cos(\varphi).\frac{1}{2}.\int_0^{2\pi/\omega}\left[\cos^2(\omega t) + \sin^2(\omega t)\right]dt$$

$$\therefore P = \frac{V_o I_o \omega}{2\pi}\cos(\varphi).\frac{1}{2}.\frac{2\pi}{\omega} = \frac{V_o I_o \cos(\varphi)}{2}$$

In the phasor representation we use:

$$\begin{aligned} V(t) &= V_o e^{j\omega t} \\ I(t) &= I_o e^{j(\omega t + \varphi)} \\ \therefore P &= \frac{1}{2}\mathrm{Re}\left[V(t).I^*(t)\right] = \frac{1}{2}\mathrm{Re}\left[V_o I_o e^{j\omega t}.e^{-j(\omega t + \varphi)}\right] \\ &= \frac{1}{2}V_o I_o \cos(\varphi) \end{aligned}$$

This is the same result as before $[\cos(-\varphi) = \cos(\varphi)]$.

Note that in the case of a capacitor (voltage lags current by $\pi/2 \Rightarrow \varphi = \pi/2$) and an inductor (voltage leads current by $\pi/2 \Rightarrow \varphi = -\pi/2$) in both situations we have $\cos(\varphi) = 0$ so there is no net power dissipation.