Answer to EM Example Q19

For normal incidence

$$\frac{E_{0R}}{E_{0I}} = \frac{(n_1 - n_2)}{(n_1 + n_2)}, \quad \frac{E_{0T}}{E_{0I}} = \frac{2n_1}{(n_1 + n_2)} \quad \text{[lectures]}$$

$$R = \left(\frac{E_{0R}}{E_{0I}}\right)^2, \quad T = \left(\frac{n_2}{n_1}\right)\cdot\left(\frac{E_{0T}}{E_{0I}}\right)^2 \text{ and } R + T = 1 \text{ [lectures]}$$

We have $R = T = \frac{1}{2}$ for solution which can be obtained in various ways using $R = T$, $R = \frac{1}{2}$, $T = \frac{1}{2}$.

Using $R = \frac{1}{2}$ we have:

$$\frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} = \frac{\left(\frac{n_1}{n_2} - 1\right)^2}{\left(\frac{n_1}{n_2} + 1\right)^2} = \frac{1}{2}$$

$$\Rightarrow 2\left(\frac{n_1}{n_2} - 1\right)^2 = \left(\frac{n_1}{n_2} + 1\right)^2$$

$$\therefore 2\left(\frac{n_1}{n_2}\right)^2 - 4\left(\frac{n_1}{n_2}\right) + 2 = \left(\frac{n_1}{n_2}\right)^2 + 2\left(\frac{n_1}{n_2}\right) + 1$$

$$\therefore \left(\frac{n_1}{n_2}\right)^2 - 6\left(\frac{n_1}{n_2}\right) + 1 = 0$$

Solving quadratic, $\left(\frac{n_1}{n_2}\right) = 3 \pm \sqrt{8} \approx \underline{5.82843}$ or $\underline{0.171573}$

IF one of $n_1, n_2 = \epsilon_{rel}^{\frac{1}{2}} = \sqrt{9} = 3$ then the possibilities are:

$\underline{n_1 = 3}$ $\quad n_2 = \frac{3}{5.82843} = 0.514719$ not appropriate For dielectric as $n < 1$

or $n_2 = \frac{3}{0.171573} = \underline{\underline{17.4853}}$

$\underline{n_2 = 3}$ $\quad n_1 = 3 \times 5.82843 = \underline{\underline{17.4853}}$ not appropriate For dielectric as $n < 1$

or $n_1 = 3 \times 0.171573 = 0.514719$

So, solutions are: $\underline{n_1 = 3, n_2 = 17.4853}$ or vice-versa