Answer to EM example Q18

The Fresnel eqn. result obtained in lectures with E polarised normal to the plane of incidence was:

$$\frac{E_{0R}}{E_{0I}} = \frac{\frac{n_1}{\mu_1}\cos\theta_I - \frac{n_2}{\mu_2}\cos\theta_T}{\frac{n_1}{\mu_1}\cos\theta_I + \frac{n_2}{\mu_2}\cos\theta_T} \qquad (*)$$

For these (non-conducting) materials $k^2 = \omega^2\mu\epsilon$ so

$$n = \frac{ck}{\omega} = c\sqrt{\mu\epsilon} = \sqrt{\mu_{rel}\epsilon_{rel}}$$

As $\epsilon_{rel1} = \epsilon_{rel2}$ (both materials have same permittivity) $(*)$ reduces to:

$$\frac{E_{0R}}{E_{0I}} = \frac{\frac{\cos\theta_I}{\sqrt{\mu_{rel1}}} - \frac{\cos\theta_T}{\sqrt{\mu_{rel2}}}}{\frac{\cos\theta_I}{\sqrt{\mu_{rel1}}} + \frac{\cos\theta_T}{\sqrt{\mu_{rel2}}}}$$

μ_{rel1}, μ_{rel2} are the relative permeabilities of medium 1, medium 2 resp.

There will be no reflected wave if the numerator in this expression $= 0$

ie if $\cos\theta_I - \sqrt{\frac{\mu_{rel1}}{\mu_{rel2}}}\cos\theta_T = 0 \qquad (+)$

In this case (because the permittivities are identical) we have that

$$\frac{n_1}{n_2} = \sqrt{\frac{\mu_{rel1}}{\mu_{rel2}}} = \frac{\sin\theta_T}{\sin\theta_I} \qquad \text{(Snell's Law)}$$

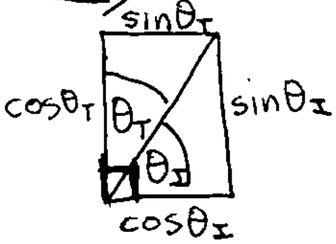
Thus $(+)$ can be written as

$$\cos\theta_I - \frac{\sin\theta_T}{\sin\theta_I}.\cos\theta_T = 0$$

$\therefore \cos\theta_I\sin\theta_I - \sin\theta_T\cos\theta_T = 0$ (provided θ_I, $\sin\theta_I \neq 0$)

$\Longrightarrow \sin(\theta_I - \theta_T)\cos(\theta_I + \theta_T) = 0$ (expand this if you don't believe it!)

$\Longrightarrow$ no reflected wave if $\theta_I + \theta_T = \frac{\pi}{2}$



No reflected wave when

$$\frac{n_1}{n_2} = \frac{\sin\theta_T}{\sin\theta_I} = \frac{\cos\theta_I}{\sin\theta_I} \Longrightarrow \tan\theta_I = \frac{n_2}{n_1}$$