Answer to Electromagnetism Example Question 17

We can use the result that $H = \frac{En}{c\mu}$, $E = \frac{c\mu H}{n}$ to rewrite 1) as

$$\frac{c\mu_1 H_{OI}}{n_1} + \frac{c\mu_1 H_{OR}}{n_1} = \frac{c\mu_2 H_{OT}}{n_2} \qquad 3)$$

To find $\frac{H_{OR}}{H_{OI}}$ we rearrange 3) to give

$$H_{OT} = \frac{n_2\mu_1}{n_1\mu_2}\left(H_{OI} + H_{OR}\right)$$

which can then be substituted in 2) leading to

$$H_{OI}\cos\theta_I - H_{OR}\cos\theta_I = \frac{n_2\mu_1}{n_1\mu_2}\left(H_{OI} + H_{OR}\right)\cos\theta_T$$

After rearrangement,

$$H_{OI}\left[\cos\theta_I - \frac{n_2\mu_1}{n_1\mu_2}\cos\theta_T\right] = H_{OR}\left[\cos\theta_I + \frac{n_2\mu_1}{n_1\mu_2}\right]$$

$$\therefore \frac{H_{OR}}{H_{OI}} = \frac{\cos\theta_I - \frac{n_2\mu_1}{n_1\mu_2}\cos\theta_T}{\cos\theta_I + \frac{n_2\mu_1}{n_1\mu_2}\cos\theta_T} = \frac{\frac{n_1}{\mu_1}\cos\theta_I - \frac{n_2}{\mu_2}\cos\theta_T}{\frac{n_1}{\mu_1}\cos\theta_I + \frac{n_2}{\mu_2}\cos\theta_T}$$

To find $\frac{H_{OT}}{H_{OI}}$ we rearrange 3) to give

$H_{OR} = \frac{n_1}{c\mu_1}\left(\frac{c\mu_2}{n_2}H_{OT} - \frac{c\mu_1}{n_1}H_{OI}\right)$ which can then be substituted in 2) leading to

$$H_{OI}\cos\theta_I - \left[\frac{n_1\mu_2 H_{OT}}{\mu_1 n_2} - H_{OI}\right]\cos\theta_I = H_{OT}\cos\theta_T$$

After rearrangement,

$$H_{OI}\left[\cos\theta_I + \cos\theta_I\right] = H_{OT}\left[\cos\theta_T + \frac{n_1\mu_2}{n_2\mu_1}\cos\theta_I\right]$$

$$\therefore \frac{H_{OT}}{H_{OI}} = \frac{2\cos\theta_I}{\cos\theta_T + \frac{n_1\mu_2}{n_2\mu_1}\cos\theta_I} = \frac{2\frac{n_2}{\mu_2}\cos\theta_I}{\frac{n_1}{\mu_1}\cos\theta_I + \frac{n_2}{\mu_2}\cos\theta_T}$$