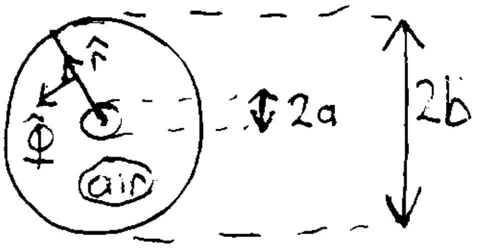
Answer to EM Example Q16

X-section through cable looks like

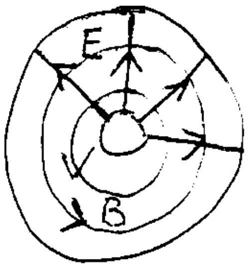


with z in direction out of sheet.

"Lines" of E are in the radial direction, $\hat{r}$

"Lines" of B are in the direction of $\hat{\phi}$, circulating around the central wire

$\rightarrow$



The Poynting vector $\underline{S} = \underline{E} \times \underline{H} = \frac{1}{\mu_0} \underline{E} \times \underline{B}$ $(B = \mu_0 H)$ gives the value of the power/area and is clearly always in the direction of $\hat{r} \times \hat{\phi} = \hat{\underline{z}}$, along the cable, out of the sheet, in this case

The time-averaged power/area

$$\underline{S}_{av} = \frac{1}{2} \text{Real} (\underline{E} \times \underline{H}^*) = \frac{1}{2} \frac{V^2 \hat{\underline{z}}}{\mu_0 r^2 c \ln^2\left(\frac{b}{a}\right)}$$

Integrating over the X-section to obtain the total power, P, we have

$$P = \int \underline{S}_{av} \cdot d\underline{a} = \frac{1}{2} \frac{V^2}{\mu_0 \ln^2\left(\frac{b}{a}\right) c} \int_a^b \frac{2\pi r \, dr}{r^2}$$

[$\underline{S}_{av}$, $d\underline{a}$ parallel, $da = 2\pi r\,dr$]

$$= \frac{1}{2} \frac{V^2 \cdot 2\pi}{\mu_0 \ln^2\left(\frac{b}{a}\right) c} \cdot \ln\left(\frac{b}{a}\right)$$

$$\therefore \quad P = \frac{\pi V^2}{\mu_0 c \ln\left(\frac{b}{a}\right)} \simeq \frac{V^2}{120 \ln\left(\frac{b}{a}\right)}$$