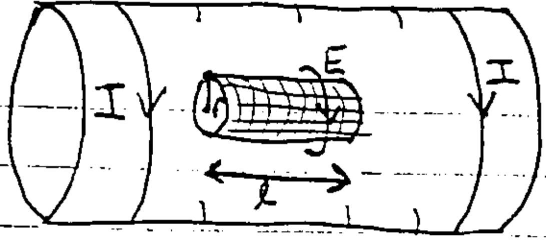
Example Q15

Answer



$\rightarrow B(t) = \mu_0 H(t)$

B assumed uniform (no end effects)

Consider the imaginary cylinder aligned along axis of solenoid as shown. For any x-section $\perp^{er}$ to axis we have from ME②

$$\int (\nabla \times \underline{E}) \cdot d\underline{a} = -\frac{\partial}{\partial t} \int \underline{B} \cdot d\underline{a} = -\pi r^2 \frac{\partial B}{\partial t}$$

Stokes $\downarrow$ theorem

$$\oint \underline{E} \cdot d\underline{l} = -\pi r^2 \frac{\partial B}{\partial t}$$

Symmetry $\Rightarrow$ E is constant around circular loop (and in dir$^{\underline{n}}$ shown if B, for example, is decreasing with time). Hence

$$2\pi r E = -\pi r^2 \frac{\partial B}{\partial t} \quad \therefore \quad E = -\frac{r}{2}\frac{\partial B}{\partial t} = -\frac{\mu_0 r}{2}\frac{\partial H}{\partial t}$$

Using Poynting vector approach the energy/time flowing out of the imaginary cylinder is

$$P = \int_{\substack{\text{closed} \\ \text{surface} \\ \text{of cylinder}}} \underline{S} \cdot d\underline{a} = \int (\underline{E} \times \underline{H}) \cdot d\underline{a}$$

Now $\underline{E}$ and $\underline{H}$ are $\perp^{er}$ to each other and are constant in magnitude over shaded section of cylindrical surface. Also, $\underline{E} \times \underline{H}$ is (anti) parallel to $d\underline{a}$ according to whether B is (increasing) decreasing with time, therefore

$$P = EH \cdot 2\pi r l = -\frac{\mu_0 r}{2}\frac{\partial H}{\partial t} \cdot H \cdot 2\pi r l$$

$$= -\mu_0 H \frac{\partial H}{\partial t} \, (\pi r^2 l)$$

[+ve if H decreasing]

$\nwarrow$ Volume of small cylinder

[Note - no contribution to integral from end sections where $\underline{S}$ and $d\underline{a}$ are perpendicular to each other]

Using the energy density expression the total energy within the cylinder is $W = \frac{1}{2}\mu_0 H^2 \cdot (\text{volume})$ and $\frac{dW}{dt} = \mu_0 H \frac{\partial H}{\partial t} \cdot (\text{volume})$

Thus $\frac{dW}{dt} = -P$ and the loss of energy within the volume correctly agrees with that flowing out of surface