

Answer to Electromagnetism Example Question 13

Now $\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} = -i\omega \underline{B}$ assuming the usual time dependence for \underline{B} .

$$\begin{aligned}\text{Thus } \underline{B} &= \frac{i}{\omega} \nabla \times \underline{E} \\ &= \frac{i}{\omega} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} E_o e^{i(\omega t - kz)} \\ 0 \\ 0 \end{pmatrix} \\ &= \frac{i}{\omega} \begin{pmatrix} 0 \\ -ikE_o \\ 0 \end{pmatrix} e^{i(\omega t - kz)}\end{aligned}$$

Therefore, with $\underline{B} = \underline{B}_o e^{i(\omega t - kz)}$, we have that

$$\underline{B}_o = \begin{pmatrix} 0 \\ \frac{kE_o}{\omega} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{(1-i)E_o}{\omega\delta} \\ 0 \end{pmatrix} = e^{-i\pi/4} \begin{pmatrix} 0 \\ \frac{\sqrt{2}E_o}{\omega\delta} \\ 0 \end{pmatrix} = e^{-i\pi/4} \begin{pmatrix} 0 \\ E_o \\ 0 \end{pmatrix} \left(\frac{\mu\sigma}{\omega} \right)^{1/2}$$

and consequently \underline{E} leads \underline{B} (and \underline{H}) by $\pi/4$.