Answer to EM example question 12

In order to be considered a good conductor at this frequency we require that

$$\sigma \gg \epsilon\omega$$

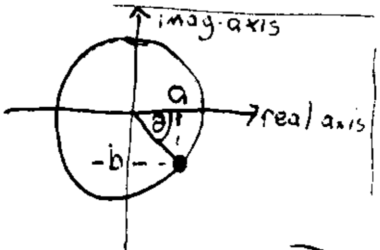
$1{\cdot}6\ \Omega^{-1}m^{-1}$ $\quad$ $51{\cdot}6 \times 2\pi \times 1800 \times 10^{6} \times 8{\cdot}85 \times 10^{-12} = 5{\cdot}16\ \Omega^{-1}m^{-1}$

This is clearly <u>not</u> satisfied, nor is the reverse, and therefore we should retain both terms in the general relationship $\quad k^2 = \omega^2\mu\epsilon - i\mu\sigma\omega$

$$= a - ib$$

(inserting values) $\quad = (7{\cdot}34 - i \times 2{\cdot}27) \times 10^4\ m^{-2}$

$$= re^{-i\theta}$$



$$\therefore\ k = r^{1/2} e^{-\frac{i\theta}{2}} \text{ or } r^{1/2} e^{-\frac{i(\theta+2\pi)}{2}}$$

$$= \pm r^{1/2} e^{-\frac{i\theta}{2}}$$

where $\quad r = \sqrt{a^2+b^2} = 7{\cdot}68 \times 10^4\,m^{-2}$

$\qquad\quad \theta = \tan^{-1}\left(\frac{b}{a}\right) = 17{\cdot}18^\circ \quad (\equiv 0{\cdot}3 \text{ rads})$

[May just use e.g. calculator to compute this directly]

$$\Longrightarrow \quad k = \pm 277\left[\cos\left(\frac{17{\cdot}18^\circ}{2}\right) - i\sin\left(\frac{17{\cdot}18^\circ}{2}\right)\right] m^{-1}$$

$$\underline{k = \pm(274 - 41{\cdot}4\,i)\ m^{-1}}$$

Given that above result is that for the standard plane-wave form

$$\underline{E}, \underline{B}, \underline{H} \ \propto\ e^{i(\omega t - kz)}$$

with z in the direction of propagation we have that the spatial decay of $\underline{E}\ \propto\ \underline{\underline{e^{-41{\cdot}4z}}}$ [clearly the exponentially <u>increasing</u> solution is unphysical]

With $z = 3\text{cm} = 0{\cdot}03\text{m} \Longrightarrow E$ decays by factor of $1/e^{-41{\cdot}4\times0{\cdot}03} \approx 3{\cdot}46$ ie to $\approx 0{\cdot}29 \times 25\ V/m \approx \underline{\underline{7{\cdot}2\ V/m}}$

<u>Note</u>: using an inappropriate $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$ good conductor expression would give $\underline{\delta = 0{\cdot}94\text{cm}}$ and a conseqvent over-estimate of the decay of E to $\approx 1V/m$