Answer to EM example question 10

$$k^2 = (k_r - ik_i)^2 = (k_r^2 - k_i^2) - i.2k_rk_i = \omega^2\mu\epsilon - i\mu\sigma\omega$$

Equating imaginary components gives

$$k_i = \frac{\mu\sigma\omega}{2k_r}$$

Equating real components

$$k_r^2 - k_i^2 = \omega^2\mu\epsilon$$

$$\Rightarrow \quad k_r^2 - \frac{\mu^2\sigma^2\omega^2}{4k_r^2} = \omega^2\mu\epsilon$$

$$\therefore \quad k_r^4 - \omega^2\mu\epsilon k_r^2 - \frac{\mu^2\sigma^2\omega^2}{4} = 0$$

Solving quadratic in k_r^2 gives

$$k_r^2 = \frac{\omega^2\mu\epsilon \pm \sqrt{\omega^4\mu^2\epsilon^2 + \mu^2\sigma^2\omega^2}}{2}$$

In practice, term in $\sqrt{}$ $> \omega^2\mu\epsilon$ and thus as k_r is defined to be real $\Rightarrow -\sqrt{}$ not allowed so we are left with $+\sqrt{}$ solution.

Re-arrangement then gives

$$k_r^2 = \omega^2\mu\epsilon\left[\frac{1 + \sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}}}{2}\right]$$

$$V_{ph} = \frac{\omega}{k_r} = \sqrt{\frac{\omega^2}{k_r^2}} = \frac{1}{(\mu\epsilon)^{1/2}}\left[\frac{2}{1 + \sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}}}\right]^{1/2}$$ as required

Clearly for a <u>poor conductor</u> with $\omega\epsilon \gg \sigma$ this reduces to $V_{ph} = \frac{1}{(\mu\epsilon)^{1/2}}$ as expected.

For a <u>good conductor</u> with $\sigma \gg \omega\epsilon$

$$V_{ph} \rightarrow \frac{1}{(\mu\epsilon)^{1/2}}\left[\frac{2}{\frac{\sigma}{\omega\epsilon}}\right]^{1/2} = \left(\frac{2\omega}{\mu\sigma}\right)^{1/2}$$ as expected