Dielectrics Examination questions 2004

SHORT QUESTIONS [2 marks for each correct answer]

a) Write down the mathematicalform and make a sketch of the expected change of polarisation with time, $P(t)$, due to the reorientation of initially randomly oriented dipoles following the application of a constant electric field.

b) If the piezoelectric coefficient of a particular material is 0.5×10^{-2} m^2C^{-1}, what is the expected magnitude of the electric field produced in response to an applied stress of $4.5\times10^6 Nm^{-2}$?

c) Give a brief description of the physics underlying the response of a pyroelectric material to an increase in temperature.

LONG QUESTION

Explain, briefly, the physical basis of the electronic polarisation mechanism. [2 marks]

The induced ionic dipole moment of a molecule in response to an applied electric field, $E = E_0 e^{i\omega t}$, is $p = -qx$ where x is the solution of

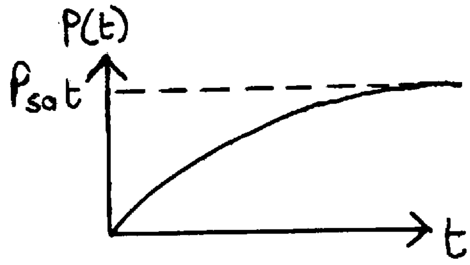
$$\frac{d^2x}{dt^2} + A\frac{dx}{dt} + Bx = -CE.$$

A, B, C and q are positive constants and ω is the angular frequency. If there are N molecules per unit volume, obtain expressions for the real and imaginary components of the frequency dependent susceptibility $\chi(\omega) = \chi_r(\omega) = i\chi_i(\omega)$. [6 marks]

In practice, to within an order of magnitude, what value would you expect for $\sqrt{B}$? [2 marks]

ANSWERS

SHORT QUESTIONS



a) Mathematically, we expect

$$P(t) = P_{sat}\left(1 - e^{-t/\tau}\right)$$

where P_{sat} is the saturation polarisation and τ is the relaxation time (which is a characteristic measure of the time that it takes for a dipole to reorient). The dipoles are initially randomly oriented giving a net zero average polarisation, but as t increases they tend to orient with the field so P then rises to some saturation value, P_{sat}, dependent on the temperature.

b) Ignoring the signs of the stress, T, and electric field, E, we have that $|E| = g\,|T|$ where g is the piezoelectric coefficient. Thus we obtain $|E| = 0.5\times10^{-2}\times4.5\times10^{6}\ \mathrm{NC^{-1}} = 2.25\times10^{4}\,\mathrm{Vm^{-1}}$.

c) For the sake of argument consider a dipole subject to increased temperature, *T*.

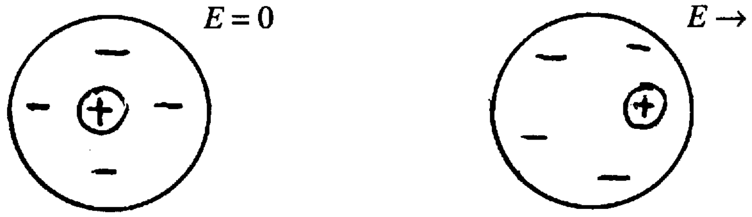
Increased *T* implies increased probability of finding electrons in higher energy states within the dipole.

These higher energy states have a different charge distribution.

This in turn implies that the dipole moment for a single dipole, and hence the polarisation for a system of such dipoles, will change. This is the basis of the pyroelectric effect.

LONG QUESTION

The electronic polarisation mechanism is the result of the rearrangement of the electron charge in an atom in response to an applied E-field.



With $E = E_0 e^{i\omega t}$ and assuming $x = x_0 e^{i\omega t}$ insertion into the given equation leads to:

$$\left(-\omega^2 + i\omega A + B\right)x = -CE.$$

$$\therefore x = \frac{-CE}{\left[\left(B-\omega^2\right)+i\omega A\right]} \quad , \quad p = -qx = \frac{qCE}{\left[\left(B-\omega^2\right)+i\omega A\right]}$$

The polarisation is then $P = Np = \dfrac{NqCE}{\left[\left(B-\omega^2\right)+i\omega A\right]} = \varepsilon_0 \chi E$ (by definition).

Thus we obtain $\chi(\omega) = \chi_r(\omega) - i\chi_i(\omega) = \dfrac{NqC}{\varepsilon_0} \cdot \dfrac{1}{\left[\left(B-\omega^2\right)+i\omega A\right]}$.

Multiplying by $1 = \dfrac{\left[\left(B-\omega^2\right)-i\omega A\right]}{\left[\left(B-\omega^2\right)-i\omega A\right]}$ this leads to the result that

$$\chi_r(\omega) - i\chi_i(\omega) = \frac{NqC}{\varepsilon_0} \cdot \frac{\left[\left(B-\omega^2\right)-i\omega A\right]}{\left[\left(B-\omega^2\right)^2+\omega^2 A^2\right]}$$ from which we can immediately extract

$$\chi_r(\omega) = \frac{NqC}{\varepsilon_0} \cdot \frac{\left(B-\omega^2\right)}{\left[\left(B-\omega^2\right)+\omega^2 A^2\right]} \quad , \quad \chi_i(\omega) = \frac{NqC}{\varepsilon_0} \cdot \frac{\omega A}{\left[\left(B-\omega^2\right)+\omega^2 A^2\right]}.$$

In the case of the ionic contribution we know that $B \equiv \omega_0^2$ where ω_0 is the natural ionic vibration frequency. This is typically in the IR region and thus $\sqrt{B} = \omega_0 \approx 10^{12} \rightarrow 10^{13}\,\mathrm{s}^{-1}$, the usual regime for ionic vibrations.